Physica A 495 (2018) 463–474

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# The complexity of the HANG SENG Index and its constituencies during the 2007–2008 Great Recession

G. Argyroudis *, F. Siokis

*University of Macedonia, Greece*

## HIGHLIGHTS

- Investigating Multifractality of Hang Seng and two sub-indices by means of MFDMA during Great Recession.
- Strong evidence of Multifractality, with the Finance Index to exhibit greater degree and more complexity for the after period phase.
- Time evolution analysis reveals strong variation in time and an enhancement of multifractal spectrum during the extreme event.

## ARTICLE INFO

*Article history:*
Received 21 July 2017
Received in revised form 11 November 2017
Available online 27 December 2017

*Keywords:*
Multifractal detrended moving average
Financial crisis
Multifractality
Complexity

## ABSTRACT

We apply the multifractal detrended moving average (MF-DMA) procedure to the daily data from HANG SENG Index (HSI) and two sub-indices, the Properties Index which consists of 10 Real Estate Companies and the Finance Index with 12 companies respectively. Two major events are considered: the 2007 and the 1997 crises. Based on scaling exponents and the singularity spectrum analysis, we show that both events reveal multiscaling and the results are robust across different indices. Furthermore, by dividing the data into two equal sub-samples for prior and after the crisis periods, we reveal that for the 2007–2008 crisis, the complexity of the HSI and Properties index remain the same between periods, while for the Finance Index, the after crisis period exhibits richer multifractality and higher complexity. Especially for the Properties Index, the results indicate that the Real Estate sector was not affected as much, by the transitory shocks of the Great Recession. As for the 1997 event, the HS Index is impacted greatly in the after period crisis exhibiting higher degree of multifractality and heterogeneity.



## 1. Introduction

The 2007 sub-prime mortgage crisis triggered a chain reaction in the U.S. and global financial systems, causing financial distress and global recession. Not only the U.S. financial markets were suffered tremendous losses but shocks were transmitted violently to global financial markets. Equity markets across the globe were among the first to absorb these shocks and some of them even crashed. This "contagion" effect was studied extensively of how shocks are transmitted across borders.

The aim of this article is to investigate the behavior of the equity market after the transmission of the financial shock and to understand the changes in the statistical characteristics of the stock exchange index under periods of extreme events.

* Corresponding author.
*E-mail address:* gargyroudis@uom.edu.gr (G. Argyroudis).

https://doi.org/10.1016/j.physa.2017.12.104
0378-4371/

We consider the Hong Kong Stock exchange market (HKEX) as an ideal laboratory for such an investigation, since is one of the largest stock exchanges in terms of capitalization and had exhibited large volatility fluctuations.[1]

Stock market exchanges follow a complex architecture, having non-linear properties and the stylized facts call for long-memory, fat tails and multifractality [1–8]. Knowing that complex systems such as the stock exchange market reveal the structure better when they are under stress [9], like the extreme events, our attempt is to study the stock market structure and their associated statistical properties during anxious periods. The importance of the stock exchange general index is well known, since it is considered as a forward looking index, indicating the future direction of the economy. For the analysis of the recent crisis, which is called the Great Recession, we utilize the general stock market index and two important constituencies that play a pivotal role in the economy: the Properties and Finance indices. For comparison purposes we analyze the 1997 period as well. In the summer of 1997 two events almost simultaneously took place: the sovereignty transfer of Hong Kong and the effects of the Asian currency meltdown caused by Baht's tremendous currency depreciation. These events prompted us to analyze the statistical properties and to provide helpful insights into underlying complexity of the stock exchange system.

Thus far, modest efforts have been made to understand the statistical properties and the effects of rare events on stock exchange markets. Drozdz et al. [10] analyzed 30 companies of DAX index, within a period of 11 years, and observed that draw downs are always accompanied by a sizeable separation of one strong eigenstate of the correlation matrix which, at the same time, reduces the variance of the noise state. The draw ups on the other hand turn out to be more competitive. In this case the dynamics spread more uniformly over the eigenstates of the correlation matrix resulting in an increase of the total information entropy. Therefore, increases are more competitive, and less collective, and thus more nonlinear correlated than decreases. Oswiecimka et al. [11] analyzed periods of upward and downward trends of the DAX index and show that multifractal spectra are broader during bull market than during its bear phase where bear market is more persistent than the bull market irrespective of the sign of fluctuations. On the foreign exchange market, Oh et al. [12] analyzed the multifractal spectra of various currencies with respect to the U.S. dollar, during the turbulent period from 1991 until 2005. They discover that the return time series show multifractal spectrum features for all four cases and especially after the Asian crisis, some currencies experienced a significant increase in multifractality. Schmitt et al. [13] considered the scaling and multifractal properties of the Chinese currency against the US dollar and the euro. They have shown that both foreign exchange rates possess multifractal properties. By dividing the time series into several samples to reveal any statistical differences, they report a change in the power spectra during the pegged system, and also, after the end of the pegged system, when the value of the exchange rate was steadily decreasing, fluctuations was still scaling with multifractal exponents very far from the previous ones.

Wang et al. [14] investigated the yuan exchange rate index after China's exchange rate system reform on the 21st July 2005. By dividing the time series into two parts according to the change in the yuan exchange rate regime in July 2008, they compare the statistical properties of the yuan exchange rate index during these two periods. They report that the change in China's exchange rate regime gave rise to the different multifractal properties of the yuan exchange rate index in these two periods, and thus has an effect on the effective exchange rate of the yuan after the exchange rate reform on the 21st July 2005. Siokis [15] analyzed the multifractal character and nonlinear properties of four major stock market indices during financial meltdowns. They consider three distinct U.S. based financial crises: the Black Monday, the Dot-Com and the Great Recession. The results show that all indices exhibit strong multifractal properties and the complexity of the markets is higher under the Black Monday event revealed by the width of the singularity spectrum and the larger $\alpha_0$ parameter. More recently Cao and Zhang [16] focused on the comparative analysis of extreme values in the Chinese and American stock markets and report that the range of extreme value of Dow Jones Industrial Average is smaller than that of Shanghai composite index, and the extreme value of Dow Jones Industrial Average is more time clustering. Also based on the multifractal detrended cross-correlation analysis algorithm they find that extreme events have nothing to do with the cross-correlation between the Chinese and American stock markets. Also, Hasan, and Salim [17] used MF-DFA technique to investigate the multifractal structure of the US and seven Asian stock markets during the crisis period. By dividing the sample into three sub-periods: pre-crisis period, crisis period and post-crisis period, they find that the markets of the US, Japan, Hong Kong, S. Korea and Indonesia show very strong non-linearities and some of them exhibit increased long range correlations of large fluctuations in index returns. Lastly, Stošić et al. [18] using the MF-DFA technique analyzed two different periods of currency exchange regimes, i.e. managed and independent floating currency rates, for eight countries. They reported a changing multifractal spectrum during the free-floating regime, an indicator of an increase in market efficiency.[2]

In terms of MF-DMA, few papers have utilized this multifractal technique such as Y. Wang et al. [20] investigating the multifractal behavior of the US dollar (USD) exchange rates. They report that twelve exchange rate series are multifractal and the major source of multifractality is long-range correlations of small and large fluctuations. They also find evidence that extreme events play an important role in the contributions to multifractality for the USD/EUR exchange rate. Also, Zhou et al. [21] applied the MF-DMA technique to investigate and compare the efficiency and multifractality of China Securities Index 300 (CSI 300). They report that the CSI 300 market becomes closer to weak-form efficiency after the introduction of CSI 300 future. They also find that the CSI 300 is featured by multifractality and there are less complexity and risk after the CSI 300 index future was introduced.

[1] In July 2016 the HKEX introduced the so-called Volatility Control Mechanism (VCM) and Closing Auction in an attempt to curb substantial increases or decreases in volatility and prices. According to various reports, volatility in HSI during the closing period can be as much as six times greater than in other developed market indices. Also the HKSE is Asia's second largest stock exchange in terms of market capitalization and the fifth largest in the world.

[2] There are other methods utilized in explaining stock market behavior before and after a financial crisis. For instances network analysis [19].

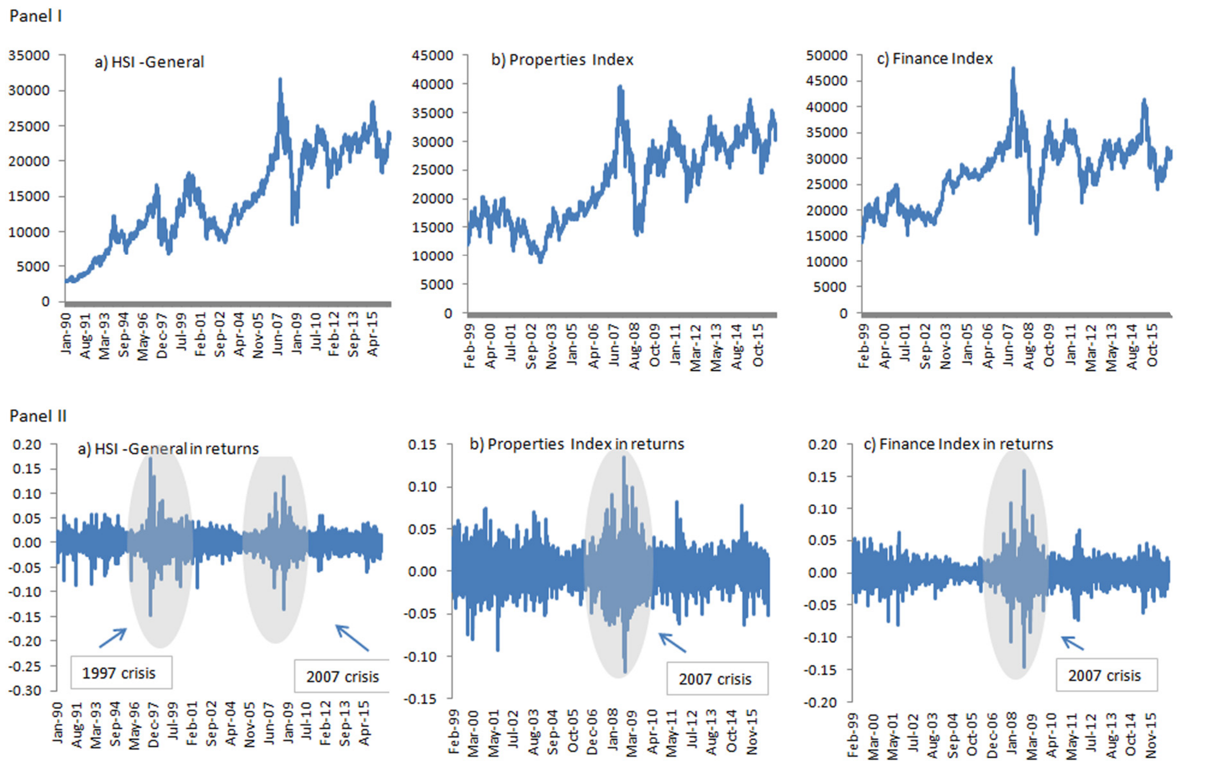


**Fig. 1.** The Hang Seng Index and two constituents, the Properties and Finance Indices. In Panel 1, HSI, Properties and Finance Indices in levels. Panel 2, all Indices in returns. Shaded areas depict the crisis.

## 2. Data and methodology

In this paper, we examine the non linear features and market complexity of the impact of 2007 world crisis on the Hong Kong Stock Exchange. We utilize three indices, namely the General Hang Seng Index, the Properties and the Finance sub-Indices, in order to investigate the statistical characteristics before and after the outbreak of the crisis. The Hang Seng Index ("HSI"), publicly launched on 24 November 1969, is the most widely quoted indicator of the performance of the Hong Kong stock market with its constituent securities grouped into Finance, Utilities, Properties, and Commerce and Industry Sub-indices. The Properties sub-index contains 10 company stocks for the Real Estate area, while the Finance Sub-index constitutes of the stocks of 12 financial companies.

The data are taken from Bloomberg and consist of end of day daily returns, and the sample period spans from August 1, 2002 to February 28, 2013 (Fig. 1), while for the 1997 event the sample starts from June 2, 1992 until February 24, 2003. Subsequently we divide the sample into two equal sub-samples, into pre and post crisis periods, with December 3, 2007 to be the cutting point. The selection of this particular dividing point is based on the National Bureau of Economic Research (NBER) announcement that the U.S. economy had entered into a recession, which by the way, lasted until June 2009. For the 1997 event we split the sample in October 17, 1997 where the index, for that particular day, lost 33.4% of its value. Following, we proceed with the analysis of the two constituents, namely the Properties and the Finance indices, but due to data restriction we perform the analysis for the 2007 event only. The division of data into pre and post crisis periods comes from the concept of bubble creation and burst. The bubble creation starts with a boom phase where stock prices are increasing and consequently the system accumulates energy and the burst is the phase where prices, after a strong shock start behaving violently resembling to a certain degree, an aftershock seismic sequence. The sub-periods under investigation for the 2007 financial crisis, might seem long, but in this case and particularly for the 2007 crisis, seems necessary to include almost 5 years of data, of each phase, in order to analyze and underline properly the importance of those two highly volatile and anxious periods in the global economy, and the significance of their effects.[3] Also our decision is supported by the Federal Reserve's low interest rate policy that began in 2001, and by 2003 the FF rate reached, at that point, the lowest historical interest rate level, in an attempt of reducing the cost of the debt stimulated and to strengthen the demand for housing. Indeed based on the expansionary monetary policy, the growth in residential prices was marked at 4.8% in 2002 and over

3 Sornette and Cauwels [22], state that the creation of the real estate bubble started in 2003 and was burst in 2007, but at the same time a cascade of bubbles were created lasting little longer. An early warning of the bubble creation was given by Zhou and Sornette [23].

**Table 1**
Descriptive Statistics: Mean, standard deviation, skewness and kurtosis for all samples and sub-sets.

| Crisis | Mean | Std deviation | Skewness | Kurtosis |
|---|---|---|---|---|
| 2007 whole sample | 0.00030 | 0.0158 | 0.0438 | 12.3320 |
| -prior period | 0.00077 | 0.0111 | −0.0491 | 5.4757 |
| -after period | −0.00017 | 0.0194 | 0.1052 | 10.3218 |
| Properties whole sample | 0.00033 | 0.0194 | 0.2089 | 7.0617 |
| -prior period | 0.00083 | 0.0159 | 0.2883 | 5.1792 |
| -after period | −0.00018 | 0.0224 | 0.2029 | 6.6998 |
| Finance whole sample | 0.00018 | 0.0162 | 0.0838 | 15.5569 |
| -prior period | 0.00058 | 0.0093 | 0.0349 | 7.0785 |
| -after period | −0.00022 | 0.0209 | 0.1207 | 10.9191 |
| 1997 whole sample | 0.00017 | 0.0182 | 0.0480 | 11.1134 |
| -prior period | 0.00063 | 0.0149 | −0.3689 | 6.1309 |
| -after period | −0.00029 | 0.0210 | 0.2276 | 11.0598 |

8.4% in 2003 (Bureau of Economic Analysis statistics). For the benefit of comparison, the same time length is adopted for the 1997 crisis as well.

The daily returns of the stock market indices calculated as $r_t = \ lnp_t \ -lnp_{t-1}$ where $p(t)$ is the price of the index on day $t$ and $r$ is the rate of return. The multifractal concept is used as a feature of the financial complex systems and we are investigating the multifractal properties of the indices based on the periods of high financial stress as depicted by the HSI (Fig. 1). Also descriptive statistics of the data, in terms of mean standard deviation, skewness and kurtosis are depicted in Table 1.

Numerous procedures are proposed to estimate the fractal dimension or the multifractal spectral of a signal. A main technique is the Multifractal Detrended Fluctuation Analysis (MF-DFA) developed by Kantelhardt et al. [24], which reduces noise effects, removes local trends and avoids spurious detection of correlations that are artifacts of nonstationarities in the time series. The work by Kantelhardt is a generalization of DFA [25] and based also on the works of Silva & Moreira and Weber & Talkner [26,27]. Motivated by DFA, Podobnik and Stanley [28] also proposed a detrended cross-correlation (MF-DCC) analysis which is designed to investigate power-law cross-correlations between different simultaneously recorded time series in the presence of nonstationarity detrending approach. Along those lines is the Multifractal Detrended Moving Average (MF-DMA) method, introduced by Gu and Zhou [29], extending Carbone's DMA method [30–32]. The MF-DMA procedure is designed to remove the local trends by subtracting the local means and to analyze multifractal time series and surfaces.[4] Research papers comparing the two methods i.e., the MF-DFA and the MF-DMA concluded that the MF-DMA procedure is more robust than the MF-DFA [29,34].

The two methods differ in the first three steps, while the MF-DMA can be described as follows.

Consider one time series $a\,(t)$, $t = 1, 2, \ldots, N$, where $N$ is the length of the series. We construct the new sequence of cumulative sums

$$x\,(t) = \sum_{i=1}^{t} a\,(i)\,. \tag{1}$$

Then, calculate the moving average function of the series of cumulative sums in a moving window [1],

$$\tilde{x}\,(t) = \frac{1}{n} \sum_{\kappa=-\lfloor (n-1)\ \theta \rfloor}^{\lceil (n-1)\ (1-\theta) \rceil} x\,(t-k)\,, \tag{2}$$

where, $n$ is the window size, $\lfloor x \rfloor$ is the largest integer not larger than $x$, $\lceil x \rceil$ is the smallest integer not smaller than $x$, and $\theta$ is the position parameter varying from 0 to 1. Hence, the moving average functions of two series consider the past $\lceil (n-1)\,(1-\theta) \rceil$ data points and the future $\lceil (n-1)\,\theta \rceil$ data points. In Gu and Zhou [29], the authors consider three special cases. The first case $\theta = 0$ refers to the backward moving average, in which the moving average functions are calculated from the past $n-1$ data points. The second case $\theta = 0.5$ refers to the centered moving average, in which the moving average functions are calculated from half past data points and half future data points. The third case $\theta = 1$ corresponds to the forward moving average, where functions are calculated from future $n-1$ data points. In this paper, we only consider the first case, the backward moving average, $\theta = 0$, since it is evident that it gives the most accurate estimation of the exponents.

Detrend the series by subtracting the moving average functions, and obtain the residual sequence $\varepsilon(i)$ using the equations as,

$$\varepsilon\,(i) = x\,(i) - \tilde{x}\,(i)\,, \tag{3}$$

where, $n - \lfloor (n-1)\,\theta \rfloor \leq i \leq N - \lfloor (n-1)\,\theta \rfloor$.

[4] A variant of the MFXDFA algorithm, termed multifractal detrending moving-average cross-correlation analysis (MFXDMA) introduced by Jiang and Zhou (2011) [33], which combines the ideas of MFDMA and DCCA. The MFXDMA technique adopts local moving average as the trend function.

And then, divide the residual sequences into $N_n$ non-overlapped segments with the equal length $n$, where $N_n = \lfloor N_n - 1 \rfloor$. Each segment can be written as $\varepsilon_\upsilon(i) = \varepsilon(l+i)$ for $1 \le i \le n$, respectively, where $l = (\upsilon - 1)n$. Then we can define the fluctuation variance as,

$$F_\upsilon^2(n) = \frac{1}{n}\sum_{i=1}^{n} \varepsilon_\upsilon^2(i). \tag{4}$$

Next, calculate the $q$th order fluctuation function using the equation:

$$F_q(n) = \left\{\frac{1}{N_n}\sum_{\upsilon=1}^{N_n} F_\upsilon^q(n)\right\}^{\frac{1}{q}}, \tag{5}$$

for $q \neq 0$. Where, $F_q(n)$ is the $q$th order fluctuation function and $F_\upsilon^q(n) = \left(F_\upsilon^2(n)\right)^{\frac{q}{2}}$.

When $q = 0$, according to L'Hôspital's rule, we have

$$ln\left[F_0(n)\right] = \frac{1}{N_n}\sum_{\upsilon=1}^{N_n} ln\left[F_\upsilon(n)\right]. \tag{6}$$

Lastly, for different values of segment length $n$, we have the power-law relationship,

$$F_q(n) \sim n^{h(q)}. \tag{7}$$

Here, the generalized Hurst exponent $h(q)$ can be obtained by observing the slope of log–log plot of $F_q(n)$ versus $n$ through the method of ordinary least squares (OLS). If $h(2) > 0.5$, the correlations are persistent (positive). An increase is likely to be followed by another increase. If $h(2) < 0.5$, the correlations are anti-persistent (negative). An increase is likely to be followed by a decrease. If $h(2) = 0.5$, the time series display a random walk behavior. The conclusion of multifractality can be obtained from the dependence of $h(q)$ on the values of fluctuation orders $q$.

The analytical relationship between generalized Hurst exponents based on MF-DMA and the Renyi exponent $\tau(q)$ is,

$$\tau(q) = qh(q) - 1. \tag{8}$$

Via a Legendre transform, another important variable set $\alpha - f(a)$ is defined by

$$\alpha = h(q) + qh'(q), f(\alpha) = q[\alpha - h(q)] + 1. \tag{9}$$

Here, $\alpha$ is the Holder exponent or singularity strength which characterizes the singularities in a time series. The singularity spectrum $f(\alpha)$ describes the singularity content of the time series.

## 3. Multifractal results

In an attempt to assess the multifractality of the indices for the two stock market events, namely the 2007 and 1997 we first calculate the fluctuation functions, $F_q(n)$ with the scaling parameter ranging from $10 \le n \le N/4$, where $N$ is the total length of the time series. Fig. 2 shows the log–log plots of the fluctuation functions $F_q(n)$ versus the time scale $n$ when $q = \pm 5$, $\pm 3$, $\pm 1$, 0 for all indices. Clearly from Fig. 2, above the crossover region, the $F_q(n)$ functions are straight lines and the slopes changes slightly when going from high positive moments to high negative moments.

Another way to characterize multifractality is to present the scaling function $\tau(q)$ in the range of $-5$ to 5. Panel I Fig. 3 depicts both the original and shuffled data derived from the power law relation between $\tau(q)$ and $q$. By shuffling the data we remove all temporal correlations in an attempt to check the amplitude distribution. We proceed by averaging over ten randomly shuffled versions of the original time series. In panel I we show that for $\tau(q)|_{q<0}$ both actual and shuffled series are almost similar, but they differ when $\tau(q)|_{q>0}$. This means that the actual and shuffled series are similar for small fluctuations, but not for large fluctuations.

Next, we convert $q$ and $\tau(q)$ to $\alpha$ and $f(\alpha)$ by a Legendre transform as $f(a) = q[a - h(q)] + 1$, $a = h(q) + qh'(q)$, where *f(α)* is the fractal dimension of the time series. The singularity spectrum quantifies the long-range correlation properties of a time series.[5] Panel II, Fig. 3 depicts the multifractal spectra $f(\alpha)$ of the original and shuffled data. The results indicate that all stock market indices, for both events exhibit high degree of multifractality. In terms of the shuffled data, where all temporal correlations have been removed, surprisingly, the width of the spectra for the HS and the Finance Indices of the 2007 event remains wide, indicating that contribution of a broad probability distribution in the observed multifractality cannot be excluded. In contrast to Gu and Zhou [29], Zhou [38] confirmed that fat-tailed distribution is the main source of multifractality, while the temporal structure has less impact in multifractality.[6] Furthermore, Hasan and Salim [17] report wide singularity spectrum for the Hong Kong stock market shuffled time series, for the 2007–2008 financial crisis, a result that Zhou, Dang and Gu [21] supported as well when they investigate the Chinese stock market for the same time period.

5 Authors in order to make a quantitative characterization of multifractal spectra, they proceed by fitting the spectrum to a quadratic function [35–37] around the position of its maximum at $\alpha_0$.

6 According to [39,40] this outcome could be caused by the finite size of the time series, resulting in an artificial multifractal behavior.

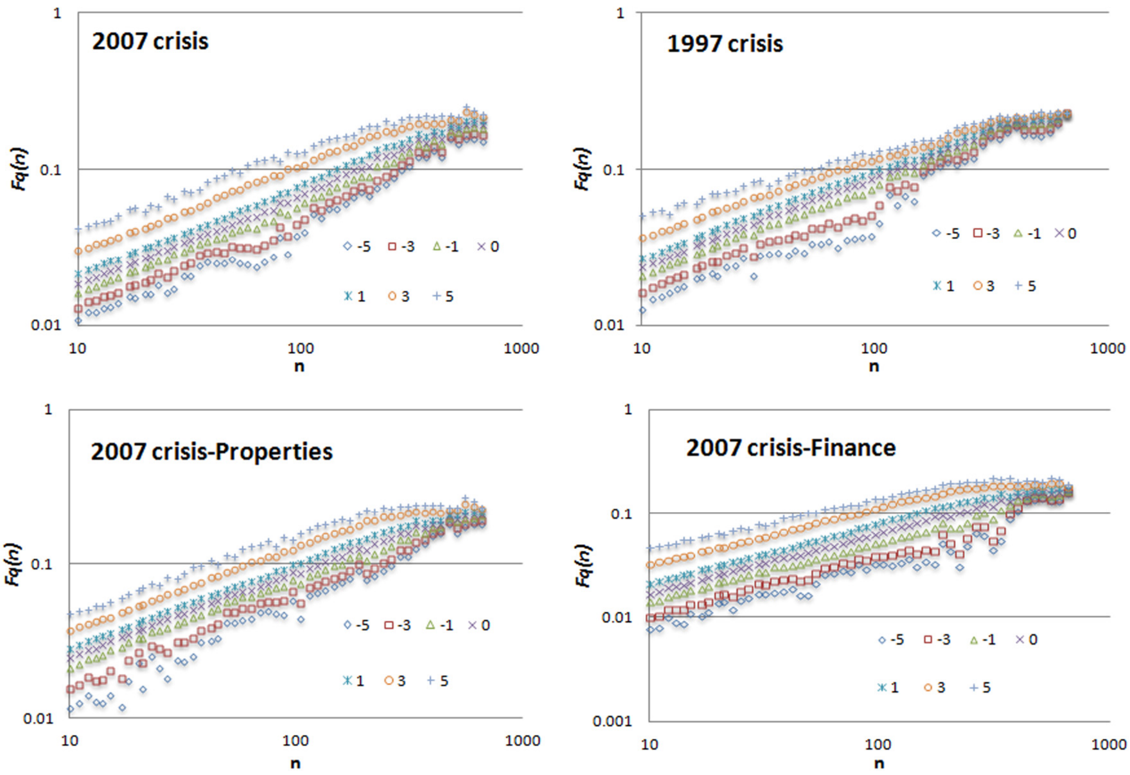


**Fig. 2.** The MF-DMA functions $Fq(n)$ versus the time scale $n$ in log–log plot.

## *3.1. The partition for prior and after-crisis periods*

Next we ask how the particular events of the 1997 and 2007 influence the market complexity. In an attempt to determine changes in market complexity we divide the series into two equal sub-periods, each comprises of 1320 trading days. Panels I and II, Fig. 4 depict the multifractal scaling exponents and the fractal dimensions. The first panel illustrates the scaling functions for pre and after the crisis periods, where the exponent $\tau(q)$ grows nonlinearly with $q$, exhibiting different behavior for $q < 0$ and $q > 0$: clearly a characterization for a multifractal set. Panel II plots the singularity spectra for the two different periods. For HSI, the value of maximum of $\alpha_{0,}$ for both phases is almost the same, localized in the vicinity of $\alpha_0 = 0.56$, implying, that fluctuations are equally persistent for both periods. In addition, the fractal dimension of the after crisis period seems marginally lower than the width of the prior phase. In essence, the width of the singularity spectrum of the after crisis period is $\Delta\alpha = 0.23$, while for the pre crisis is $\Delta\alpha = 0.25$. In addition, the after crisis sequence is characterized by a left-skewed singularity spectrum, indicating that there is a dominance of the larger price fluctuations, while that of the pre crisis period is quite symmetric.

In the same path, the sequence for both periods of the Properties index is multifractal with the after-crisis period to be marginally less multifractal than of the pre crisis period. Specifically, the after-crisis period characterized by $\Delta\alpha = 0.279$ while for the pre crisis $\Delta\alpha = 0.308$. But in contrast to the HSI, the singularity spectrum for the pre-crisis period is characterized by a right-skewed multifractal spectrum, indicating that there is a relative dominance of the larger generalized Hurst exponent, or that small price fluctuations dominate the process, while that of the after crisis period is quite symmetric, indicating a balance between the different magnitude effects of the price fluctuations. Based on the comparison of the degree of multifractality between the two subsets, the results suggest that the crisis, initiated in the U.S. had impacted differently the multifractality of the Properties Index. Thus, the sequence of the after crisis period is less heterogeneous than the one of the pre crisis period.

Different results emerged with the analysis of the Finance Index. The singularity spectrum of the after-crisis period reveals greater multifractal degree compared with the pre-crisis period. It seems that the complexity of the financial market had increased after the outbreak of the crisis, signifying an increase in the stock price fluctuation. Apparently, this outcome is consistent with the view that based on the high degree of integration and global interconnectedness of the financial markets, a shock to the U.S. market could immediately transmitted and impacted all other developed financial markets.

Finally, in terms of comparison to 2007, the analysis of the 1997 crisis shows that the phase occurred after the outbreak has richer multifractality and higher heterogeneity than the prior one.

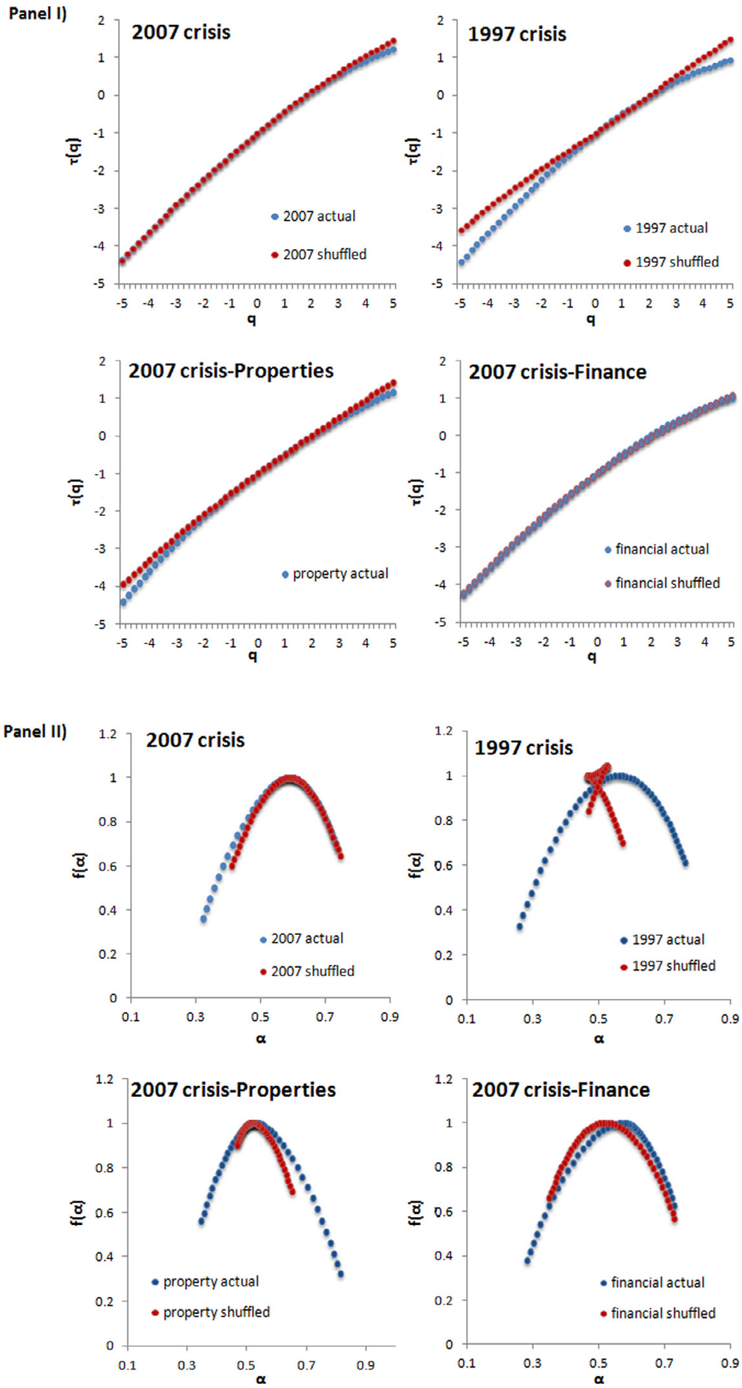


**Fig. 3.** Scale Exponents and Fractal Dimensions. Panel I depicts the Multifractal scaling exponents for both actual and shuffled data. Panel II plots the Fractal dimensions.

The findings of this section and especially the different behavior exhibited by the Properties and Finance Indices, point to some policy implications. Firstly, it is widely recognized that Real Estate bubbles are important sources of financial meltdown. Therefore, like any other asset prices, real estate prices are important elements of the monetary transmission mechanism and therefore, in terms of conducting monetary policy, Central Bank should pay attention to changes of the real estate prices. Certainly the response of monetary policy should depend mainly on the nature of the shock and the degree of permanence of that particular shock.

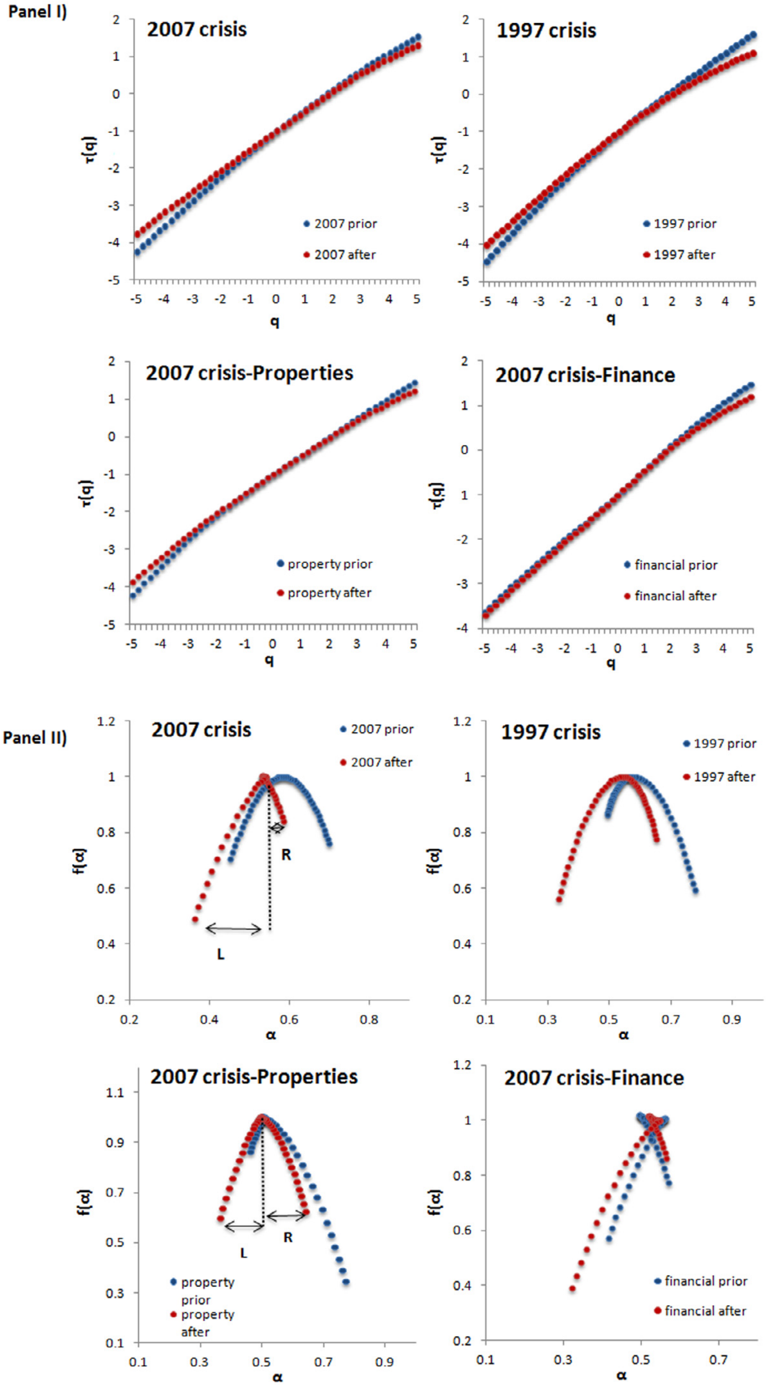


**Fig. 4.** Partition of prior and after crisis periods. Panel I depicts the Multifractal scaling exponents for pre and after period. Panel II plots the Fractal dimensions for both periods.

## *3.2. Time depended analysis*

We now turn into a time dependent analysis calculating the temporal evolution of multifractality. We utilize the sliding window of specific events methodology, keeping the time interval of each window constant to 1320 days, which is almost 5 years worth of trading days. The shift of the window is set to 22 trading days, equivalently to one business month. We calculate the maximum $\alpha_0$ and the width parameter function, with the period of the first window to be from June 1, 1992

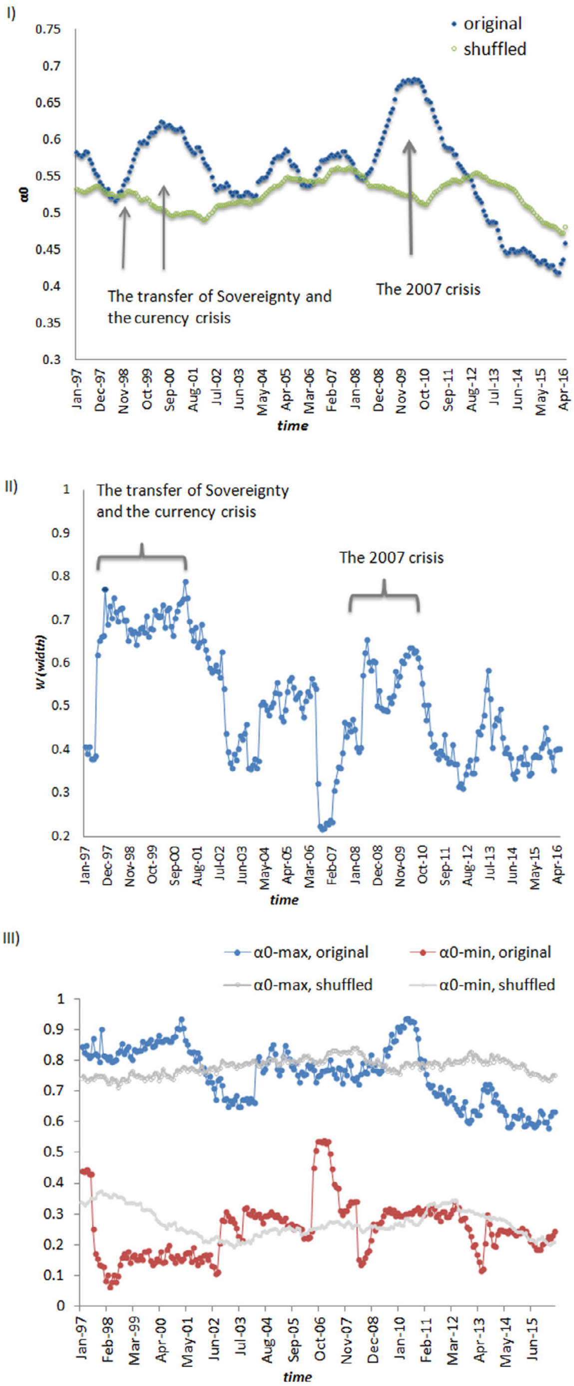


**Fig. 5.** Time variation of the Hang Seng index for the parameter $\alpha_0$, the width $W$ and the values of $\alpha_{max}$ and $\alpha_{min}$. The data span from Jan. 1992 to June 2016.

until June 30, 1997 and the last one from May 2, 2011 until June 1, 2016. We have extended the sample until June 1 2016, in an attempt to capture the dynamics of the calculated parameters.

Fig. 5 plots the variation in time of two parameters, $\alpha_0$, the value where the spectrum has its maximum, $f(\alpha) = 1$, the width of the spectrum (W) and the evolution of the $\alpha_{max}$ and $\alpha_{min}$ values. All parameters exhibit strong variability with a characterization of sharp dynamical changes, while one of the most pronounced effects on the index is the significant change, during the period of both crises. Panel I Fig. 5 depicts the fluctuation of parameter $\alpha_{0.}$ During the two crises the value of $\alpha_0$ is much higher, compared with the value of other time periods, indicating a higher local fractal dimension or larger complexity.

As for the 1997 event, the parameter starts increasing from its lowest point in January 1998 which corresponds to the window of Jan 1993 to Jan 1998 and reaches its maximum levels in April 2000, (corresponding to the window of April 1995 to April 2000) and fluctuated around that level until May 2001. In particular, the highest increase of the $\alpha_0$ value is documented right after the transfer of sovereignty over Hong Kong from the United Kingdom to the People's Republic of China on July 1st 1997 and the effect of the Asian currency crisis.

For the 2007 crisis, the $\alpha_0$ parameter starts to increase in January 2008, reaching its maximum in Dec. 2009, corresponding to the window Dec. 2004 to Dec. 2009 and stays in high levels until Oct 2010. After Oct 2010 the trend is reversed and the $\alpha_0$ value started decreasing. Finally, panel I Fig. 5 presents the variation of the shuffled data, which perform like a random walk process, taking values between 0.5 and 0.55.

Panel II Fig. 5 plots the time variation of the singularity spectrum width showing great variability during the two events. Clearly, based on the width value for the two events there is a gain of multifractality with the 1997 event depicting a greater degree of multifractality and for a longer time period. The width increases after June 1997 which corresponds to the window June 1992 to June 1997 and fluctuates in high levels until May 2001 which corresponds to the window May 1993 to May 2001. As for the 2007 crisis, the $\alpha_0$ parameter starts to increase in Oct 2007, reaching its maximum in Sep 2008 and stays in high levels until Oct 2010. After Oct 2010 the trend is reversed and the $\alpha_0$ value started decreasing.

Surprisingly, around July 2012 the width of the spectrum started increasing again for a short period of time. This increase could be attributed mainly to two events. Firstly, in June 2012, HKEX announced its cash offer to acquire the London Metal Exchange (LME), the world's premier metal exchange and the acquisition was completed in December 2012. Secondly, beginning June 2012 HKEX started the process of creating an over the counter (OTC) Clearing House, offering OTC derivatives clearing services. This process finalized in November 2013.

Lastly in panel III Fig. 5, the $\alpha_{max}$ and $\alpha_{min}$ are depicted along with the associated shuffled data. For the 1997 event, great variability is recorded both for the $\alpha_{max}$ and $\alpha_{min}$ values, indicating a change from homogeneous to heterogeneous dynamics. During that period the spectrum is "richer" in structure, with wider range of fractal dimensions. As for the 2007 event, the $\alpha_{min}$ value drops sharply at the beginning of the crisis, prior to any movement made by the $\alpha_{max}$ value.

In Fig. 6 we present the time evolution of the two sub-indices. The analysis starts from February 1999 extending it to June 2016, in an attempt of capturing the dynamics of the indices. Same pattern is depicted for the 2007 event for both Properties and Finance indices. The $\alpha_0$ parameter fluctuates between 0.48 and 0.55 until April 2008 where the parameter records the lowest value. From that point on, an increase of the $\alpha_0$ and $W$ parameters is observed in association with the main event of the crisis, indicating an enhancement of the multifractality degree. The maximum value is achieved just around Aug 2010.[7,8]

As for the degree of multifractality (Panel II), the width of the singularity spectrum appears to be time-varying and intense. In particular, the degree of multifractality is characterized by a clear increase just prior to the main shock, increasing thus the degree of market complexity. The most striking feature in the plot of width for the Finance Index is the significant and more pronounced change of the time pattern just prior to the outbreak of the crisis. In particular the width shows a sudden increase just around August 2007, where the credit crunch in the US financial markets took place. It seems that the credit crunch had global implications, since the new financial products and the packaging of asset backed securities, composed of risky mortgages were sold to banks, international investors and to pension funds worldwide. Thus, the results support the view that a shock in the US financial markets is transmitted immediately to other developed financial markets.

Lastly, Panel III depicts the temporal variation of the $\alpha_{max}$ and $\alpha_{min}$ values, for both sub-indices. The value of the $\alpha_{min}$ for the Properties Index decreases around the end of 2007, while for the Finance there is a drop in the value again in August 2007. Therefore, the degree of multifractality exhibits strong time variations, which are associated with the dynamic evolution of the major shocks during the crisis periods.

## 4. Conclusion

We investigate the complexity of two major stock market events in Hong Kong. Using the MF-DMA method, we showed the existence of multifractality in the Stock market indices, both for the 1997 and 2007 events. The results are extended also for the two major sub-indices, the property and financial indices. Shuffling the data, to remove temporal correlations, we get in two instances some degree of multifractality meaning that there is long memory in the data. Next by splitting the sample into two equal sub-samples we try to determine if market complexity changes between periods. For the 1997 event, the HS index is greatly impacted by the sequence of events, with the after crisis period to exhibit much higher degree of multifractality and consequently complexity. For the 2007 event, the degree of multifractality recorded for the HSI is almost the same for both sub-sets. Similar results derived from the analysis of the Properties Index, which could be translated that the Real Estate sector was equally affected, for both periods, by the transitory shocks of the Great Recession. On the other hand, richer multifractality was recorded for the Finance index, during the after crisis period, indicating higher complexity, a change from homogeneity to hetereogeneity, or an increase in price fluctuations. Significant finding is the capture by the data the credit crunch event commenced in August 2007. Overall, the analysis of the multifractality for the two

[7] This corresponds to the period of Aug. 2005 to Aug. 2010.

[8] Worth noting is the greater fluctuation exhibited by the $\alpha_0$ value of the Finance Index, as well as reaching a higher point than the Properties Index, during the 2007–2008 crisis.

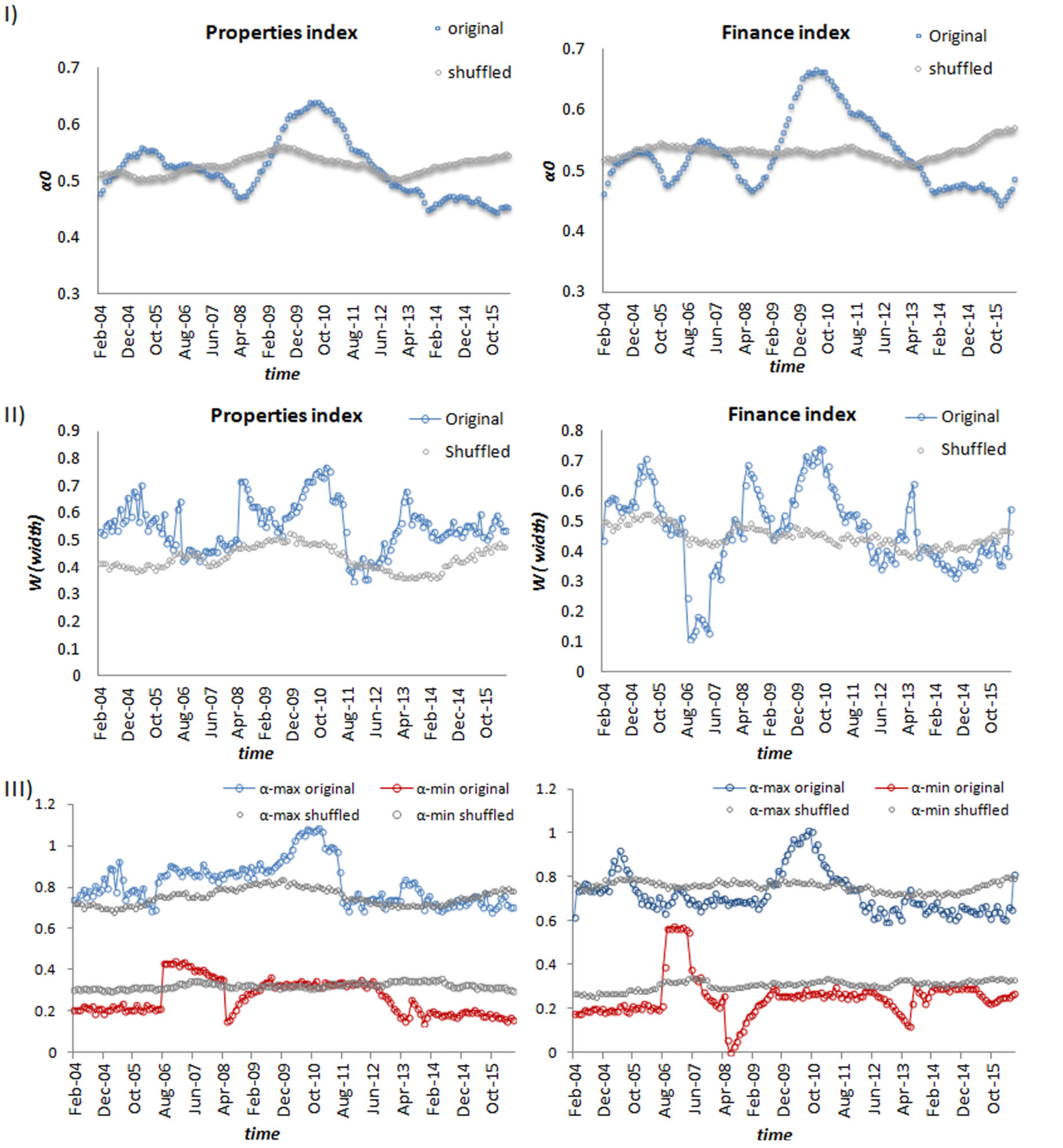


**Fig. 6.** Time variation of the Property and Financial indices for the parameter $\alpha_0$ (singularity spectrum maximum), the width $W$ and the values of $\alpha_{max}$ and $\alpha_{min}$. The data span is from Feb. 1999 to June 2016.

sub-indices suggests that the 2007 event had impacted differently the two sub-indices, with the Finance index to exhibit greater multifractality especially for the after crisis period: an outcome that someone would have expected given the large and volatile international capital flow and the increased world financial markets integration.

Finally our findings hope to enhance the understanding of the mechanisms determining the dynamics during extreme financial events and to develop diagnostic models in predicting and thus diminishing the impact of financial meltdowns.

## Acknowledgment

We would like to thank Prof. X-W. Zhou for providing the MF-DMA MATLAB code, as well as the related bibliography.

## References


[1] B.B. Mandelbrot, The variation of certain speculative prices, J. Bus. 36 (4) (1963) 394–419.
[2] B.B. Mandelbrot, A fractal walk down wall street, Sci. Amer. 298 (1999) 70.
[3] R.N. Mantegna, H.E. Stanley, Turbulence and financial markets, Nature 383 (1996) 587.
[4] M. Pasquini, M. Serva, Multiscale behaviour of volatility autocorrelations in a financial market, Econom. Lett. 65 (3) (1999) 275–279.
[5] L. Calvet, A. Fisher, Multifractality in asset returns: Theory and evidence, Rev. Econom. Statist. 84 (2002) 381–406.
[6] K. Matia, Y. Ashkenazy, H.E. Stanley, Multifractal properties of price fluctuations of stocks and commodities, Europhys. Lett. 61 (3) (2003) 422–428.
[7] Y. Ashkenazy, S. Havlin, P.Ch. Ivanov, C.-K. Peng, V. Schulte-Frohlinde, H.E. Stanley, Magnitude and sign scaling in power-law correlated time series, Physica A 323 (2003) 19–41.
[8] T. Lux, T. Kaizoji, Forecasting volatility and volume in the Tokyo Stock Market: Long memory, fractality and regime switching, J. Econom. Dynam. Control 31 (6) (2007) 1808–1843.
[9] D. Sornette, Why Stock Markets Crash? Critical Events in Complex Financial Systems, Princeton University Press, Princeton, NJ, 2003.
[10] S. Drozdz, F. Grummer, F. Ruf, J. Speth, Dynamics of competition between collectivity and noise in the stock market, Physica A 287 (2000) 440–449.
[11] P. Oswiecimka, J. Kwapien, S. Drozdz, A.Z. Gorski, R. Rak, Different fractal properties of positive and negative returns, Acta Phys. Polon. A 114 (2008) 547–553.
[12] G. Oh, C. Eom, S. Havlin, W. Jung, F. Wang, H.E. Stanley, S. Kim, A multifractal analysis of Asian foreign exchange markets, Eur. Phys. J. B. 85 (2012) 214.
[13] F. Schmitt, L. Ma, T. Angounou, Multifractal analysis of the dollar-yuan and euro-yuan exchange rates before and after the reform of the peg, Quant. Financ. 11 (4) (2011) 505–513.
[14] Dong-Hua Wang, Xiao-Wen Yu, Yuan-Yuan Suo, Statistical properties of the yuan exchange rate index, Physica A 391 (12) (2012) 3503–3512.
[15] F.M. Siokis, Financial Markets during highly anxious time: Multifractal fluctuations in asset returns, Fractals 25 (2017) 03.
[16] G. Cao, M. Zhang, Extreme values in the Chinese and American stock markets based on detrended fluctuation analysis, Physica A 436 (2015) 25–35.
[17] R. Hasan, M. Salim, Multifractal analysis of Asian markets during 2007–2008 financial crisis, Physica A 419 (2015) 746–761.
[18] D. Stošić, D. Stošić, T. Stošić, H.E. Stanley, Multifractal analysis of managed and independent float exchange rates, Physica A 428 (2015) 13–18.
[19] R. Han, W.J. Xie, X. Xiong, W. Zhang, W.X. Zhou, Market correlation structure changes around the great crash: A random matrix theory analysis of the chinese stock market, Fluct. Noise Lett. 16 (2017) 1750018.
[20] ., Y. Wang C. Wu, Z. Pan, Multifractal detrending moving average analysis on the US dollar exchange rates, Physica A 390 (2011) 3512–3523.
[21] W. Zhou, Y. Dang, R. Gu, Efficiency and multifractality analysis of CSI 300 based on multifractal detrending moving average algorithm, Physica A 392 (2013) 1429–1438.
[22] D. Sornette, P. Cauwels, 1980–2008: The illusion of the perpetual money machine and what it bodes for the future, Risks 2 (2) (2014) 103–113.
[23] W.X. Zhou, D. Sornette, Is there a real-estate bubble in the US? Physica A 361 (2006) 297–308.
[24] J.W. Kantelhardt, et al., Multifractal detrended fluctuation analysis of nonstationary time series, Physica A 316 (2002) 87–114.
[25] C.-K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Mosaic organization of DNA nucleotides, Phys. Rev. E 49 (1994) 2.
[26] A. Castro e Silva, J.G. Moreira, Roughness exponents to calculate multi-affine fractal exponents, Physica A 235 (1997) 327–333.
[27] R.O. Weber, P. Talkher, Spectra and correlations of climate data from days to decades, J. Geophys. Res. 106 (2001) D17 20, 131-20 144.
[28] B. Podobnik, H.E. Stanley, Detrended cross-correlation analysis: A new method for analyzing two nonstationary time series, Phys. Rev. Lett. 100 (2008) 084102.
[29] Gao-Feng Gu, Wei-Xing Zhou, Detrending moving average algorithm for multifractals, Phys. Rev. E. 82 (2010) 011136.
[30] E. Alessio, A. Carbone, G. Castelli, V. Frappietro, Second-order moving average and scaling of stochastic time series, Eur. Phys. J. B 27 (2002) 197–200.
[31] A. Carbone, Algorithm to estimate the hurst exponent of high-dimensional fractals, Phys. Rev. E. 76 (2007) 056703.
[32] S. Arianos, A. Carbone, Detrending moving average algorithm: A closed-form approximation of the scaling law, Physica A 382 (2007) 9–15.
[33] Z.-Q. Jiang, W.-X. Zhou, Multifractal detrending moving-average cross-correlation analysis, Phys. Rev. E 84 (2011) 016106.
[34] Y.-H. Shao, G.-F. Gu, Z.-Q. Jiang, W.-X. Zhou, D. Sornette, Comparing the performance of FA, DFA and DMA using different synthetic long-range correlated time series, Sci. Rep. 2 (2012) 835.
[35] Y. Shimizu, S. Thurner, K. Ehrenberger, Multifractal spectra as a measure of complexity in human posture, Fractals 10 (2002) 103–116.
[36] Y. Ying, Z. Xin-tian, Multifractal description of stock price index fluctuation using a quadratic function fitting, Physica A 387 (2008) 511–518.
[37] D. Stosic, D. Stosic, T. Stosic, H.E. Stanley, Multifractal properties of price change and volume change of stock market indices, Physica A 428 (2015) 46–51.
[38] W.-X. Zhou, The components of empirical multifractality in financial returns, Europhys. Lett. 88 (2009) 28004.
[39] L. Czarnecki, D. Grech, Multifractal dynamics of stock markets, Acta Phys. Polon. A 117 (2010) 623–629.
[40] Z.-Q. Jiang, W.-X. Zhou, Multifractality in stock indexes: Fact or fiction? Physica A 387 (2008) 3605–3614.