



# Spillover effects of Great Recession on Hong-Kong's Real Estate Market: An analysis based on Causality Plane and Tsallis Curves of Complexity–Entropy

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## HIGHLIGHTS

- Investigating the Hong Kong's housing market complexity during Great Recession.
- Changes in informational efficiency without Kowloon area being affected by the crisis.
- Time evolution analysis reveals periods of predictable underlying dynamics.

## ARTICLE INFO

### Article history:

Received 15 October 2018

Received in revised form 20 December 2018

Available online 30 April 2019

### Keywords:

Permutation entropy

Sub-prime loan crisis

Complexity–entropy causality plane

Tsallis complexity–entropy curve

Informational efficiency

## ABSTRACT

This paper investigates the impact of the sub-prime loan crisis on the Real Estate Market of Hong-Kong. Based on permutation entropy, complexity–entropy causality plane and Tsallis complexity–entropy curve, we characterize the complexity of the housing indices—both in terms of size and region—and distinguish the level of informational efficiency. By calculating the quantifiers we report that most indices exhibit a behavior equivalent to a persistent stochastic dynamics with Hurst exponents between 0.5 and 0.7. The outbreak of the crisis had changed the dynamical structure of the indices decreasing the level of randomness and increasing considerably their regularity and predictability. Only the index of the Kowloon area seems not impacted by the crisis, exhibiting higher levels of informational efficiency. The results are robust based on the utilization of two different entropy definitions: The Shannon and Tsallis- $q$  entropy. Lastly, with the temporal evolution of the indices, we identify periods where the underlying dynamical structure of the market was impacted by certain events like the SARS epidemic and the imposition of Special Stamp Duty on housing.

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## 1. Introduction

The 2007–2008 financial crisis in the U.S.A. caused an unprecedented international turmoil. The substantial growth both in mortgage credit and in housing prices initiated an increase in mortgage delinquencies, as interest rates increased sharply. The burst of the housing bubble and the meltdown of the associated prices triggered a chain of reaction to other financial markets, both domestically and internationally. Asset prices across the globe fell, while financial volatility rose substantially. Among the markets, where prices plummeted considerably, were the housing markets and particularly those that historically have exhibited higher volatility.

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The aim of this paper is to investigate the effects of the U.S. Real Estate collapse on other housing markets and particularly on Hong-Kong area. The rationale for testing the particular region is that the Hong-Kong housing market could be considered as an ideal laboratory, based on the robust demand for properties, the relatively low interest rates environment for a long period of time and the large and frequent increases in property prices. More precisely, house prices have tripled in Hong-Kong from 2003 to 2015 while construction surged sharply.

The Real Estate sector, and particularly the housing market under stress has been much less studied compared to other economic assets, and also little is known as far as the information efficiency of Real Estate markets is concerned. Moreover, an important discourse in the discussion of the Real Estate market is the relevance of the theory of Efficient Market Hypothesis (EMH). According to EMH, efficient prices reflect all available information, and follow a rather “random walk” where it is impossible to make a profit systematically, or above average return (Fama [1,2]). Put differently, the market is able to absorb the information of yesterday’s price change, without influencing today’s price change, making thus the stock market memory-free. The well-known example in Malkiel [3] of a chimpanzee throwing darts to select a portfolio that would do as well as the portfolio of an expert displays the notion about the EMH.

Nevertheless, the intellectual dominance of the EMH and the assumption that prices are random walks was put into question, a number of times especially during anxious times. According to empirical evidence, levels of inefficiency can be found even in the most competitive markets. Shiller [4] challenged EMH and asked why an efficient stock market should be that volatile, when the actual price should have been the same as the optimal forecast? The dot-com crisis demonstrated that for a short period of time pricing irregularities and predictable patterns can take place. After all, as Grossman and Stiglitz [5] noted, prices cannot reflect all available information, because if they did, there would not have been any incentives for the informed traders to spend their resources to receive compensation.

The EMH theory was unconfirmed in the findings of numerous articles. For example, Barkoulas and Baum [6] with spectral analysis applied to stock returns of some companies included in the Dow Jones Industrial Index, found no fractal structure but some evidence of long memory. Ito and Sugiyama [7] showed that the market efficiency in US stock market is not stable but rather fluctuates through time; it was more inefficient during the late 80s, and became efficient around 2000. Moreover Di Matteo et al. [8] reported an inclination from pure Brownian motion. Eom et al. [9] report a positive relationship between the predictability of the hit rate and the degree of efficiency, concluding that the prediction of future price changes is feasible by using the Hurst exponent as a measurement. Bariviera et al. [10] studied the long varying behavior of sovereign and corporate bond markets of seven EU countries. Using the Detrended Fluctuation Analysis (DFA) and the sliding window technique, they report different memory dynamics in bond indices, after the Great Recession.

Against the theory of randomness by Bachelier [11], Zhang [12] introduced a new approach named conditional entropy, which measures possible profit pockets that could be revealed from the inefficiency margin in market dynamics. Zunino et al. [13] in an attempt to discriminate the market dynamics report that developed stock markets have lower number of forbidden patterns and higher normalized permutation entropy than the emerging ones. In Zunino et al. [14], the complexity–entropy causality plane is used to classify the stage of stock market development. The developed stock markets show higher entropy and lower permutation complexity, while the emergent stock markets behave in the opposite way, with signs of time correlation and inefficiency. Also, Zunino et al. [15] presents a complexity–entropy causality plane for 30 bond indices, a non-exhaustively studied sector. The findings confirm that permutation entropy is higher for the developed countries, and market size is correlated with permutation entropy, showing that developed and emerging bond markets differ.

Bariviera et al. [16] investigated the impact of two economic situations on the informational efficiency of European sovereign bond markets. The establishment of the common currency produced diverse informational efficiency levels among the countries and was negatively affected by the 2008 financial crisis. In Serinaldi et al. [17] the position of chaotic deterministic systems are located in the upper left region of the plane, while the stochastic and noisy signals are closed to the limit point (1, 0). Bariviera et al. [18] use the concepts of permutation entropy and permutation statistical complexity to reveal the temporal correlation of interest rates, and more specifically the Libor rate. Using sliding window to show the evolution of these quantifiers, a complex behavior is revealed, and temporal correlations in Libor rates are shown. Zunino et al. [19] analyzed the efficiency of the European corporate bond sectorial indices before and during the financial crisis of 2007. The sectors that are connected to the financial economy like financial banks, insurance, basic resources and financial services have presented lower informational efficiency. On the other hand, sectors related to the real economy like healthcare, food & beverage and utilities presented less change in the informational efficiency. In Bariviera et al. [20], the efficiency of crude oil market is questioned. The authors concluded that during important economic events the efficiency of the oil has changed. Lastly, several works applied the entropy concept to quantify the efficiency level in various financial markets, Oh et al. [21], Risso [22], Alvarez-Ramirez et al. [23], Martina et al. [24], and Ortiz-Cruz et al. [25].

In this paper we examine the evolution of the informational efficiency of the Real Estate market in Hong Kong on a size-based and on a region-based perspective during the 1998–2017 period. We want to understand the changes in the statistical characteristics of the Real Estate market during a period of anxious time. By identifying the statistical properties of the housing market indices under stress, we hope to improve our perception about the mechanism behind the dynamics of the real estate market and to develop predictive models for Real Estate price changes. We are studying the informational efficiency level, before and after the financial meltdown of 2007, by using the permutation entropy concept. Different Hong Kong price indices are used based on (a) region namely the Hong Kong Mass, Hong Kong Island, Kowloon, New Territories

East, New Territories West, and (b) size like the General Housing price index for overall flat sizes, Large Housing price index and Medium/Small Housing price index. Flats with net floor area over 1076 sq. ft. are included in the large housing price index.<sup>1</sup>

The remainder of the paper is organized as follows. In the following section we describe the information–permutation theory, in terms of Shannon and Tsallis entropies. In Section 3 we describe the data used followed by the presentation and discussion of the empirical results obtained from the different indices of the housing markets. In Section 4 we summarize the findings of the paper and conclude.

## 2. Methodology

### 2.1. Permutation entropy

Entropy is a general concept of information theory, and it is expressed in terms of a discrete set of probabilities. The estimation of the associated probabilistic distribution comprises as the first step for the calculation of the so-called entropy quantifier, for a given time series. Various methods have been suggested for the estimation of the probability distribution. These methods are, the Fourier analysis, introduced by Powell and Percival [26], the wavelet transformation by Rosso and Mairal [27], the symbolic analysis by Daw et al. [28] and the amplitude statistics (Micco et al. [29]). In our case we utilize the permutation entropy, a procedure introduced by Bandt and Pompe [30] (B&P) in order to construct a probability distribution from a time series. The B&P approach is based on the celebrated Shannon entropic measure, and several applications drawn from scientific disciplines; from medical issues, like observing the results of anesthetic drugs (Jordan et al. [31], Li et al. [32], Olofsen et al. [33], Li et al. [34]), or epileptic seizures (Cao et al. [35], Li and Richards [36], Bruzzo et al. [37]) to various uses refer to astrophysical plasmas (Weck et al. [38]), and financial markets as well. The complexity of arbitrary time series is calculated using an ordinal pattern, and then time causality is assessed from the comparison of the adjacent values. The main advantages of permutation entropy, as set forth by Zunino et al. [13] are its robustness, simplicity, its invariance considering nonlinear monotonous transformations, applicability to any type of time series, the fact that it does not require a large sample size like for instance in fractal analysis, and lastly that it provides a fast computational algorithm.

For the permutation entropy the degree of structure in a process is not quantified by randomness measures, and therefore it is necessary to obtain measures of statistical complexity. Those measures were firstly introduced by Lopez-Ruiz et al. [39], which manage to identify essential dynamics, and distinguish different degrees of periodicity and chaos. Also, the embedding dimension  $D$  defines the number of symbols that formulae the ordinal pattern, and guides for the necessary length of time series in a way that the condition  $M \gg D!$  is necessary for reliable statistics.

Given a time series  $\{x_t : t = 1, \dots, M\}$ , an embedding dimension  $D > 1$ , and a time delay  $\tau$ , we consider the ordinal patterns of order  $D$  which are generated by a segment  $s \rightarrow (x_{s-(D-1)\tau}, x_{s-(D-2)\tau}, \dots, x_{s-\tau}, x_s)$ . The ordinal pattern of the time series is the permutation  $\pi = (r_0, r_1, \dots, r_{D-1})$  of the index set  $(0, 1, \dots, D-1)$  corresponding to the ranking of the  $x$  in ascending order, namely  $x_{s-r_{D-1}\tau} \leq x_{s-r_{D-2}\tau} \leq \dots \leq x_{s-r_1\tau} \leq x_{s-r_0\tau}$ . In order to get a unique result we consider that if  $x_{s-r_i\tau} = x_{s-r_{i-1}\tau}$  then  $r_i < r_{i-1}$ . For all the  $D!$  possible permutations  $\pi_i$  of order  $D$ , their associated relative frequencies can be computed by

$$p(\pi_i) = \frac{\# \{s | 1 + (D-1)\tau \leq s \leq M \text{ has ordinal pattern } \pi_i\}}{M - (D-1)\tau} \quad (1)$$

where  $\#$  stands for frequency of occurrence of  $\pi$ . Also  $P = \{p(\pi_i), i = 1, \dots, D!\}$ . The permutation entropy is defined as the Shannon entropy of this probability distribution, i.e.,

$$S[P] = - \sum_{i=1}^{D!} p(\pi_i) \ln p(\pi_i) \quad (2)$$

### 2.2. Complexity–entropy causality plane

We get the normalized permutation entropy by dividing the Shannon entropy by  $\ln D!$

$$H_S[P] = \frac{\left[ - \sum_{i=1}^{D!} p(\pi_i) \ln(p(\pi_i)) \right]}{\ln D!} \quad (3)$$

The normalized permutation entropy lies between 0 and 1.  $H_S[P] = 1$  when there is pure random sequence and our ignorance is maximal, while  $H_S[P] = 0$  when we can predict the outcome with certainty.

The results of permutation entropy are linked to the choice of the embedding dimension. The latter depends heavily on the length  $M$  of the time series in a way that  $M \gg D!$  must be satisfied. B&P proposed for practicality to work with  $3 \leq D \leq 7$ , arguing that a  $D$  of 2 would not work properly because there are few distinct states, while a  $D > 7$  would cause

<sup>1</sup> For more information, see ([www1.centadata.com](http://www1.centadata.com)).

memory restrictions, and as Staniek and Lehnertz [40] also state, one cannot have reliable statistics about the ordering dynamics. The embedding dimension is set to  $D = 4$ , but  $D = 3$  and  $D = 5$  are used for comparison purposes. The time delay  $\tau$ , which is the time separation between symbols is set as  $\tau = 1$ .

We also search for the degree of correlational structure in the time series with the use of statistical complexity (Lopez-Ruiz et al. [39], Lamberti et al. [41], Rosso et al. [42]). A complexity measure developed is the so-called Jensen–Shannon complexity  $C_{JS}$ , which is a functional of the discrete distribution  $P$  of  $N$  probabilities associated with the time series (Martin et al. [43]). It is a range of  $C_{JS}$ 's values provided in the range of  $C_{min}$  and  $C_{max}$  given for an entropy value. Once the Jensen–Shannon complexity normalized such as  $0 \leq C_{JS} \leq 1$  then

$$C_{JS} [P] = -2 \frac{S \left[ \frac{P+P_e}{2} \right] - \frac{1}{2} S [P] - \frac{1}{2} S [P_e]}{\frac{N+1}{N} \log (N+1) - 2 \log (2N) + \log N} H [P] \quad (4)$$

Where  $S$  is the Shannon entropy,  $H[P]$  is the normalized Shannon entropy and  $P_e = \{1/N, \dots, 1/N\}$  is the uniform probability. The Jensen–Shannon complexity has been introduced in nonlinear dynamics analysis to detect essential details of the dynamics and discriminate different degrees of periodicity and chaos (Lamberti et al. [41]).

### 2.3. Tsallis- $q$ entropy

Although the Jensen–Shannon complexity is widely used, another monoparametric generalization of the statistical complexity and normalized Shannon entropy, based on the Tsallis  $q$  entropy has been recently developed.<sup>2</sup> We are able to produce a group of complexity measures and understand some of the different meanings of complexity, with the values of the parameter  $q$  in the Tsallis entropy (Ribeiro et al. [45], Martin et al. [43]). Therefore following the above works we attempt to create a series of  $q$  values that will help in constructing the so-called  $q$ -complexity entropy curve and we will compare the Tsallis causality curve ( $H_q [P]$  and  $C_q [P]$ ) with the Jensen–Shannon plane ( $H_S [P]$  and  $C_{JS} [P]$ ). The normalized Tsallis entropy is given as

$$H_q [P] = \frac{\frac{1}{q-1} \sum_{j=1}^N \left( (p_j) - (p_j)^q \right)}{\frac{1-N^{(1-q)}}{q-1}} \quad (5)$$

For a full derivation of the normalized Tsallis  $q$  and the associated complexity–entropy curve please see Ribeiro et al. [45] and Martin et al. [43] as well as Rosso et al. [42] which considers  $q = 1$ . For the Jensen complexity measure  $C_q [P]$ , divergence and  $Q_0$  are also calculated as

$$\frac{1}{2(q-1)} \sum_{j=1}^N (p_j)^q \left\{ \left( \frac{p_j + \frac{1}{D!}}{2} \right)^{1-q} - (p_j)^{1-q} \right\} + \frac{1}{2(q-1)} \sum_{j=1}^N \left( \frac{1}{D!} \right)^q \left\{ \left( \frac{p_j + \frac{1}{D!}}{2} \right)^{1-q} - \left( \frac{1}{D!} \right)^{1-q} \right\},$$

$$Q_0 = (1-q) \left\{ 1 - \left[ \frac{(1+N^q)(1+N)^{(1-q)} + (N-1)}{2^{(2-q)N}} \right] \right\}^{-1}.$$

The complexity for the Tsallis entropy is calculated as

$$C_q [P] = Q * H_q [P] \quad (6)$$

where the disequilibrium  $Q = \text{Jensen divergence} * Q_0$ .

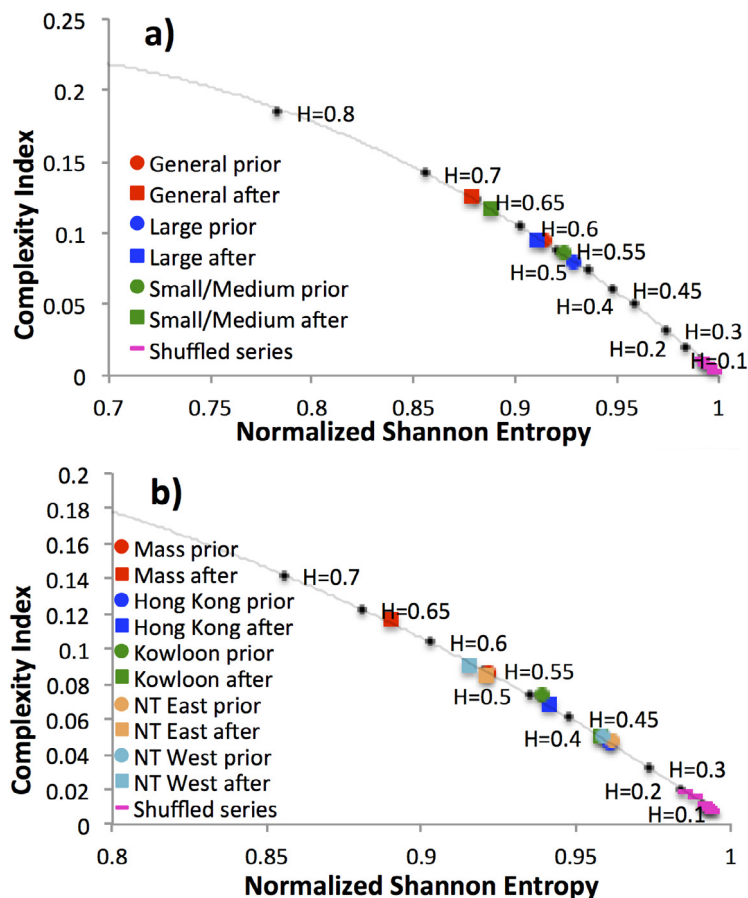
## 3. Data and empirical results

### 3.1. Data

The data are taken from the Centaline Property Agency Limited, consisting of transaction contract prices. The General Housing Price index is weekly based and called ‘‘Centa-City Leading Index’’, (CCL) reflecting the overall price trend of Real Estate market of Hong Kong. Two sub indices, called Large and Small/Medium show the trend of prices for flats over and under 1076 sq. ft. Also, we analyze the Mass index, a region based general index, and its four sub-indices: Hong Kong Island, Kowloon, New Territories East and New Territories West.

Data cover the period from April 1998 to September 2017 and divided into two equal non-overlapping segments. Each sub-period consists of 505 weekly observations, with the first segment, named pre-crisis period, spanning from April 1998 to beginning of December 2007, while the crisis/post-crisis period span from early December 2007 to September 2017. The division of the data (December 2017) is based on the announcement by National Bureau of Economic Research (NBER) that the U.S. economy officially entered in a recession, which lasted until the end of June 2009.

<sup>2</sup> For further analysis of complexity–entropy curves in the context of the Rényi entropy please see M. Jauregui et al. [44].



**Fig. 1.** Location of the various indices before and after the outbreak of the crisis, in terms of size and region in the CECP (a and b panel respectively), with embedding dimensions  $D = 4$ , and time delay  $\tau = 1$ .

### 3.2. Empirical results

The permutation entropy and the statistical complexity are calculated for embedding dimension  $D = 4$  and delay  $\tau = 1$ , based on the B&P criterion that satisfies  $M \gg D!$ . We also performed the analysis for two extra embedding dimensions  $D = 3$  and  $D = 5$ . The results seem to be independent of the embedding dimension selected for the symbolic reconstruction of the original time series, therefore we focus on  $D = 4$ . We start our analysis by presenting in Fig. 1, panel (a) and (b), the Shannon Complexity–Entropy Causality Plane (CECP) for all Real Estate indices in Hong Kong, partitioned by size and location. Panel (a) presents the calculated quantifiers of the pre-crisis period based on size indices. All indices before the crisis appear to have higher permutation entropy and lower complexity value, than the quantifiers of the post-crisis period. In particular, prior to the crisis entropy and complexity values range from 0.914 to 0.929 and 0.079 to 0.095 and after the outbreak of the crisis, the quantifiers are 0.878 to 0.91 and 0.095 to 0.126 respectively. It can be concluded that the financial crisis has affected the dynamic structure of the indices, increasing their regularity and predictability. In terms of efficiency, which according to Fama's sense means that the time series are random, the General index and the Large and Small/Medium size flats indices show a lower efficiency level, during the post-crisis period, compared with the prior period. In other words, the results for the post-crisis period suggest that last week's price influence next week's price.

Panel (b), Fig. 1, presents the analysis of the four sub-regions of Hong Kong, namely Hong Kong Island, Kowloon, New Territories East, and New Territories West. Although most indices depict the same picture as the one with the size, where the after crisis period quantifiers exhibit lower permutation and higher complexity than with the pre crisis period, Kowloon index behave much differently. Based on the quantifiers, the index for the post-crisis period exhibits higher permutation and lower complexity than the pre-crisis period, signifying that the dynamical structure of the Kowloon index is affected the opposite than the other indices. Likely, the index was not affected by the crisis, evidently by the increased informational efficiency which by the way, is higher than the level before the crisis. A possible explanation of the apparent departure could be found on the following reasons. Kowloon is a major part of the decentralized trend, due to Hong Kong Island's high prices for office and retail space, and, also, a major recipient of strong demand for residential space manifestly by the major construction projects that took place, especially after the second half of the 2000-decade.

**Table 1**  
Normalized Shannon and Tsallis permutation entropy and complexity for the Real Estate Indices in Hong Kong with  $\tau = 1$  and  $D = 4$ .

Real Estate Index	Pre crisis period				After crisis period			
	Hs Shannon	Hq Tsallis	Cjs Shannon	Cq Tsallis	Hs Shannon	Hq Tsallis	Cjs Shannon	Cq Tsallis
General	0.914	0.942	0.095	0.115	0.879	0.915	0.126	0.152
Large	0.929	0.952	0.080	0.096	0.911	0.937	0.096	0.115
Small/Medium	0.924	0.949	0.086	0.103	0.888	0.920	0.117	0.141
Mean	0.922	0.948	0.087	0.105	0.892	0.924	0.113	0.136
Standard deviation	0.008	0.005	0.008	0.009	0.016	0.012	0.016	0.019
Mass	0.922	0.947	0.087	0.105	0.890	0.923	0.117	0.140
Hong Kong	0.961	0.974	0.046	0.055	0.941	0.961	0.068	0.082
Kowloon	0.939	0.961	0.074	0.088	0.958	0.972	0.050	0.061
New Territories East	0.962	0.975	0.047	0.057	0.921	0.944	0.085	0.102
New Territories West	0.959	0.974	0.050	0.061	0.916	0.940	0.091	0.109
Mean	0.948	0.966	0.061	0.073	0.925	0.948	0.082	0.099
Standard deviation	0.017	0.012	0.018	0.022	0.026	0.019	0.025	0.030

In order to show the robustness of the previous results we present in Fig. 1 the quantifiers of the shuffled data. Please note that the quantifiers of the shuffled data are close to 1 in terms of permutation and 0 for statistical complexity, necessary conditions for a time series to be considered as a Gaussian random process (white noise). According to Fama's notion, this optimal combination (1, 0) characterizes the strong form of the EMH, where the market is strongly efficient reflecting both private and public available information. For the exact estimation of all quantifiers for each sub-period with embedding dimension  $D = 4$  and  $\tau = 1$  are detailed in Table 1.

In order to compare the results, derived by the symbolic permutation entropy, with the Hurst exponent  $h$ , we generate a time series with the use of the fractional Brownian motion. As is well known, values of  $h = 0.5$  means that the process is purely random while values of over 0.5 means that positive (or negative) steps are followed by positive (or negative) steps. If the value is less than 0.5 then a positive step is followed by a negative step and the opposite. Thus, we perform the analysis of fractional Brownian motions (fBMs) using different Hurst exponents. Although there is a wide use of correlated stochastic processes for financial time series modeling, in our case, we generate 1000 numerical independent realizations of fBMs with Hurst exponent  $h \in \{0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.80, 0.90\}$ . The length of data is  $N = 1010$  data points and we use the algorithm proposed by Abry and Sellan [46], and Bardet et al. [47].

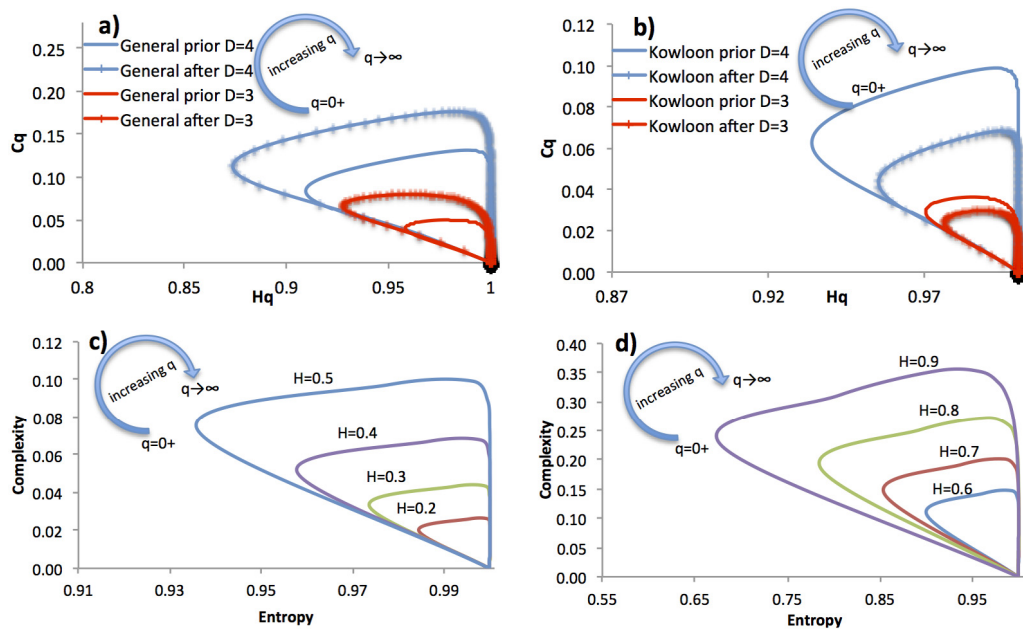
The fBMs generated curves are illustrated in Fig. 1 in both panels, along with the quantifiers from the permutation entropy method. The values of the entropy complexity plane as discussed before are very close to the ones obtained by fBMs process, meaning that they share some similar dynamical properties. The  $h$  exponents of fBMs are within the range of 0.5–0.7, and few of them close to 0.5, particularly for the pre-crisis period, a sign of uncorrelated dynamics.

Next we consider the Tsallis  $q$  entropy and complexity, and present in Fig. 2 the  $q$ -complexity-entropy curve. Using the fractional Brownian motion, following Hosking's procedure (Hosking [48]) we apply various Hurst exponents (0.2, 0.3, ..., 0.9) for the embedding dimension  $D = 3$  and  $D = 4$ . The plots are average values from 100 realizations of 1000 length time series of the entropy and complexity causality plane, with the parameter  $q$  ranging from  $q = 0+$  to  $q = 1000$  with  $10^3$  evenly spaced points between them. The  $D = 3$  and  $D = 4$  results of our Real Estate indices in Fig. 2, panel (a) and (b) resemble those of the fractional Brownian motion in the last two panels (c) and (d). All form loops with different size for the pre and after crisis period and the larger the value of  $h$  the broader the loop. Since most of the indices behave similarly as the General index, we present only the General and the Kowloon indices. More specifically, The General index depicted in Fig. 2 revealed broader loops for the after crisis period, while for the Kowloon index, the loop for the after-crisis period is much smaller, for both  $D = 3$  and  $D = 4$ . Except for the Kowloon case, the results confirm the notion that during the outbreak of the crisis, the housing market indices exhibit larger Hurst exponents.

The dependence of  $q$  values on the Hurst exponent is shown in Fig. 3. As in Ribeiro et al. [45], Hurst exponent  $h = (0, 0.1, 0.2, \dots, 1)$  is presented along with minimum value of the normalized entropy, as a function of  $q$  at  $q = q_H$ , and the maximum value of the complexity as a function of  $q$ , when  $q = q_C$ . Fig. 3 panel (a) shows for  $D = 4$  the positive correlation between  $q_H$  and the Hurst exponent, while in panel (b) the  $q_C$  increases monotonically as  $h$  increases. Panels (c) and (d) present the values of  $q$  the  $q = q_H$  and  $q = q_C$  for different embedding dimensions, i.e.  $D = 3, 4, 5$ . The extreme values  $q_H$  and  $q_C$  have similar trend, therefore we present only the General index. When  $D = 3$ ,  $q_H$  for both periods prior and after the crisis exhibit higher value followed by  $D = 4$  and  $D = 5$ , respectively. On the other hand, the values of  $q_C$  are almost the same for both indices, regardless of the embedded  $D$  value. All  $q$  values optimizing  $H_q$  and  $C_q$  are presented in Table 2.

### 3.3. Time evolution

We now investigate the time evolution of the quantifiers in an attempt to test if the underlying process changes before and after the Great Recession. Considering a 100 weeks window, which corresponds to about 500 business days (or two



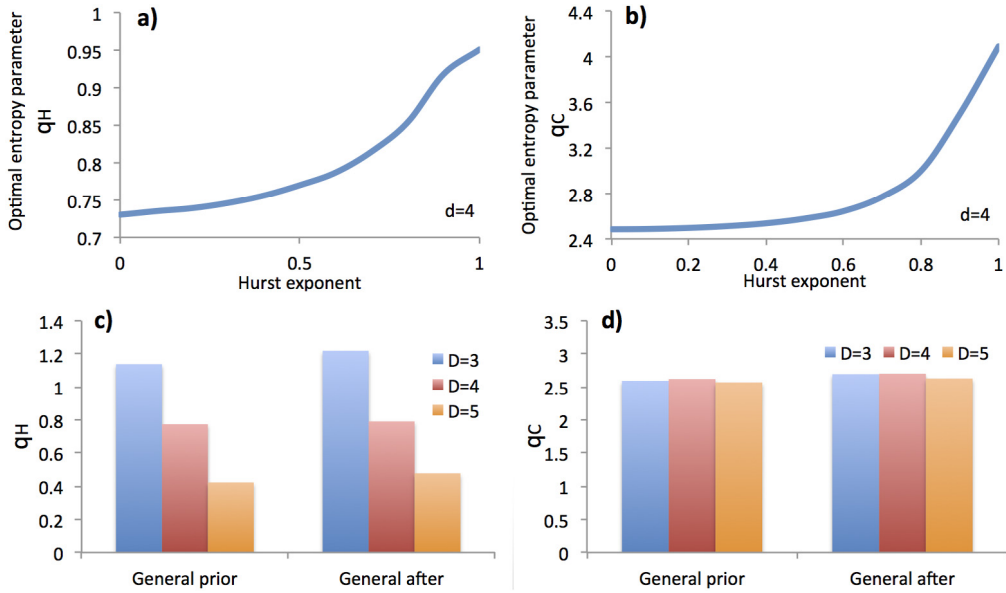
**Fig. 2.** Dependence of the entropy  $Hq$  and complexity  $Cq$  on the parameter  $q$  for all General and Kowloon indices (Panels (a) and (b)) and embedding dimensions ( $D = 3, 4$ ). The star markers indicate the points  $(Hq, Cq)$  for  $q = 0+$ , while the open circles are the same for  $q \rightarrow \infty$ . Panels (c) and (d) show the dependence of the entropy  $Hq$  and complexity  $Cq$  on the parameter  $q$  for the fractional Brownian motion with  $D = 4$  and values of the Hurst exponent  $h = (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ . The values of  $q$  are increasing (from  $q = 0+$  to  $q = 1000$  with  $10^3$  evenly spaced points between them).

**Table 2**

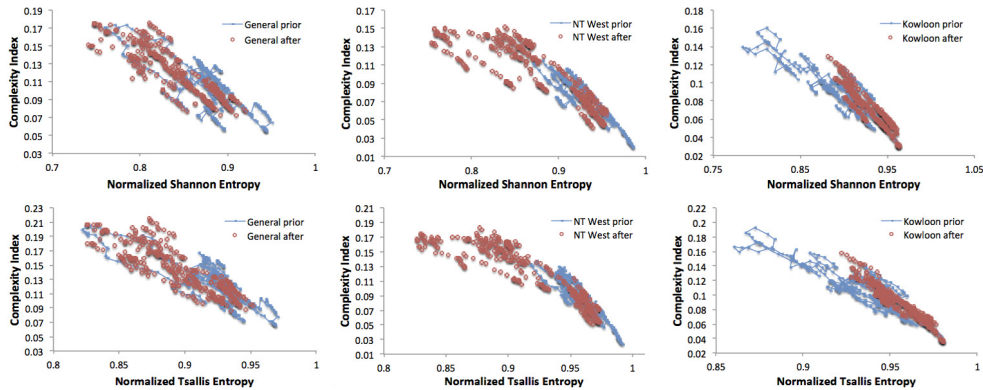
Values of  $q$  that optimize  $Hq$  and  $Cq$  for the Real Estate Indices in Hong Kong, with  $D = 3, 4, 5$ . Shaded values are for the after-crisis period for all indices.

Index	Entropy			Complexity		
	D = 3	D = 4	D = 5	D = 3	D = 4	D = 5
General prior	1.138	0.773	0.422	2.593	2.620	2.569
General after	1.219	0.790	0.477	2.694	2.701	2.628
Large prior	1.130	0.779	0.500	2.575	2.591	2.547
Large after	1.222	0.801	0.528	2.631	2.641	2.574
Small/Medium prior	1.132	0.770	0.358	2.579	2.599	2.549
Small/Medium after	1.183	0.802	0.467	2.665	2.683	2.629
Mass prior	1.127	0.784	0.464	2.584	2.603	2.557
Mass after	1.208	0.786	0.437	2.647	2.680	2.624
Kowloon prior	1.113	0.724	0.355	2.559	2.571	2.519
Kowloon after	1.084	0.770	0.478	2.542	2.537	2.508
Hong Kong prior	1.125	0.784	0.533	2.524	2.536	2.512
Hong Kong after	1.112	0.772	0.409	2.551	2.564	2.521
NT East prior	1.119	0.742	0.359	2.520	2.531	2.504
NT East after	1.274	0.811	0.379	2.638	2.622	2.548
NT West prior	1.110	0.745	0.459	2.520	2.531	2.505
NT West after	1.246	0.805	0.495	2.638	2.629	2.570

years) and shifting through the time series with a step of one week, we measure the time variation of both permutation entropy and statistical complexity. This methodology is utilized in order to detect changes in the underlying stochastic processes. For each index the sample size is divided into two periods, with the first spanning from 1998 to 2007, and the second from 2008 to 2017. Fig. 4, presents the entropy complexity causality plane for both Shannon and Tsallis methods, with  $D = 4$ , for the General, New Territories West and Kowloon indices. Although, both methods behave similarly, clearly, the entropy complexity causality plane discriminates between the two periods, as seen by the different locality of the points. For the calculation of the time evolution of the quantifiers, Kullback–Leibler and Jensen–Tsallis methods were used. Since they offer similar results, we present the Jensen–Tsallis ones. For the first two indices, the after-crisis period show lower permutation entropy and higher statistical complexity, compared to the prior-crisis period, while the opposite emerges for the Kowloon index. The Kowloon index exhibits greater degree of randomness, since the outbreak of the crisis, consistent with a totally random stochastic process.



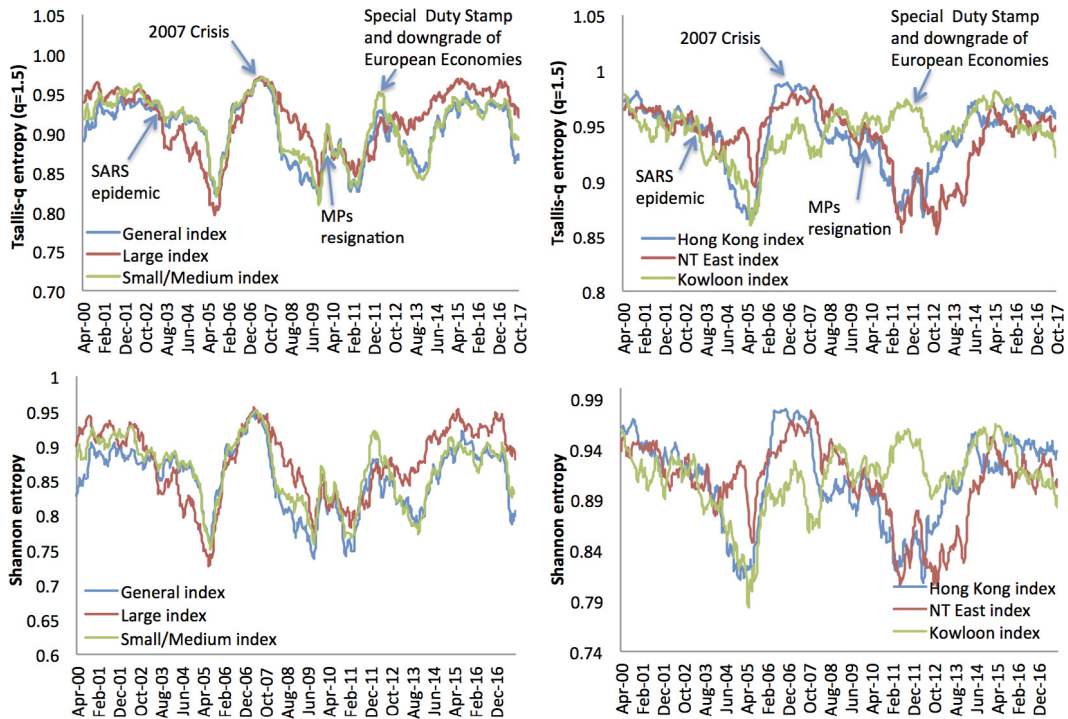
**Fig. 3.** Values  $q_H$  and  $q_C$  obtained from the simulations and the results for the fractional Brownian motion. Panel (a) shows the values of  $q = q_H$  for which  $H_q$  reaches a minimum as a function of the Hurst exponent  $h$ , while Panel (b) shows the values of  $q = q_C$  for which  $C_q$  reaches a maximum as a function of the Hurst exponent  $h$  and  $d = 4$ . Panels (c) and (d) show the extreme values  $q_H$  and  $q_C$  obtained for time series with  $D = 3, 4, 5$ .



**Fig. 4.** Complexity–entropy causality plane based on Shannon and Tsallis ( $q = 1.5$ ) entropy methods for different time periods. Movement of permutation quantifiers for selected indices with  $D = 4$ ,  $t = 1$  window.

Also Fig. 5 depicts the time variation of the permutation entropy for Shannon and Tsallis- $q$  entropy. For the sake of parsimonious and aesthetically correct figures, and since the general index gives similar results with the Large, Small/Medium, Mass, and New Territories West index, we display only the General index. On the other hand, Hong Kong and New Territories East index give similar results too. In this case we take into consideration the former one. Lastly, Kowloon index is one trend of its own. Furthermore, all Tsallis entropy results for each index for every  $q$  ( $q = 1.4, 1.45, 1.5$ ) behave in the same pattern, therefore  $q = 1.5$  is selected. Both methods exhibit similar fluctuations in permutation entropy values. The results of the two methods confirm what we have observed in the previous Fig. 4: before the crisis the entropy values are higher and around at the beginning of 2008 they start to fall significantly. The information efficiency, based on the quantifiers seems to decrease in value after the outbreak of the crisis, for most of the indices except the Kowloon index. The upward trend of the informational efficiency, since the outbreak of the crisis could be explained firstly, on Hong Kong’s government intention to transform Kowloon into a second Central Business District (CBD). Secondly, on the increasing inflow of investors’ capital from Mainland China to Kowloon housing market, due to hedging against RMB depreciation (Hui et al. [49]), and because they consider Kowloon area as a strategic gateway. Lastly, people are moving out of CBD of Hong Kong for less expensive real estate areas (CBRE 2012).

Both permutation entropy approaches seem to capture well some particular incidents that took place during the period under investigation. The first event occurred few years before the crisis, on March 2003 and had to do with the so-called severe acute respiratory syndrome (SARS) epidemic (Lam et al. [50]). After the outbreak of the epidemic all indices



**Fig. 5.** Normalized permutation entropy for Tsallis- $q$  and Shannon, for  $D = 4$  and  $\tau = 1$ . The arrows indicate the beginning of an event for the representative housing indices.

declined in terms of permutation entropy value for more than two years. Another event generating lower entropy value, magnifying the impact of the financial crisis was the resignation of five Hong Kong members of the Parliament, in January 2010, demanding more civil liberties. Finally, the last decrease in permutation entropy values occurs in the first quarter of 2012, where a Special Stamp Duty on Hong Kong housing was imposed and the rating agencies announcement of the sovereign downgrade of nine European economies, (including France, Austria, Spain, and Italy).

#### 4. Conclusion

In this paper we investigate the behavior of Hong Kong's housing market indices prior and after the 2007 U.S. financial crisis. Based on normalized permutation entropy and statistical complexity we calculate the informational efficiency level. We found that the financial crisis modified the degree of randomness and the complexity of the housing indices. In all cases but one, the indices, measured in terms of size and region, exhibit lower permutation entropy and higher complexity, meaning that the financial crisis event affected the dynamical structure of the indices, increasing their regularity and predictability. But the Kowloon index quantifiers exhibit higher permutation entropy value denoting greater informational efficiency or less regularity and loss of complexity. This outcome seems to be robust by utilizing the Tsallis- $q$  entropy methodology.

In addition, by taking a 100-week long window, shifting through the time series with a step of one week, we calculate the time variation of the permutation entropy for representative indices. Both, Shannon and Tsallis- $q$  entropy methods display very similar trends and values. It seems that they capture well the impact of the financial meltdown on Hong-Kong's housing market indices, but with a small time delay. Again, based on time variation the Kowloon index depicts a different trend with the quantifiers to increase their values even during the outbreak of the crisis. Lastly, the permutation entropy captures well the other major events such as the SARS epidemic, the Eurozone crisis and the resignation of the house representatives.

#### Acknowledgments

We would like to acknowledge the careful work and constructive criticism of the referees.

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